Principles of Design

Phi, The Golden Rectangle, and Fibonacci

by Boyd Holtan

Beginning smiths are quickly faced with design issues, soon after they have developed the basic blacksmithing forge skills. Since most are interested in ornamental items, the design questions become very important in producing the most attractive and appealing products. There are some very old ideas that may be of interest to these smiths.

If you were asked to design a rectangular sign, what would you choose for the length and width in order to have the most pleasing appearance? Many believe that the most pleasing rectangle is the "Golden Rectangle" with a length about 1 .618 times the width. Using that formula, if you wanted a sign 5 feet wide, it should be 5 times 1.618 or 8.09 feet long. There are many rectangles we sec in use that are very close to Golden Rectangles. A couple of examples are playing cards and the American Flag.

Back in 1876, Gustav Fechner, a German psychologist, asked many people to choose the most pleasing rectangle from a set of rectangles. He found that over 3/4 of the people chose rectangles very similar to thc Golden Rectangle! Fechner's study has sincc been repeated at least three timcs with similar results.

How is the Golden Rectangle made? Let's start with square ABCD (see Fig. 1) and cut side AB in half at point E. Then draw the diagonal EC in the second half of the square (Fig. 2). Add the length of that diagonal to the side of the first half-square. AE +

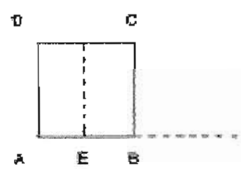

Fig 1

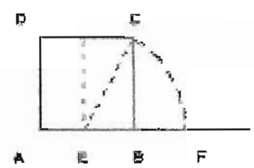

Fig. 2

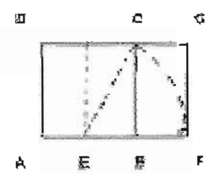

Fig. 3

EC = AF. Now, erect a perpendicular at F and complete the rectangle AFGD (Fig. 3). Rectangle AFGD is the Golden Rectangle.

How do we know that this is the Golden Rectangle? If we let the original square be I unit on each side, the right triangle EBC has legs of 1/2 and 1 (EB and BC). If we use the Pythagorean Theorem, we will find that the diagonal
EC is

$$\frac{\sqrt{5}}{2}$$

We can thcn find the length of the rectangle AF by adding the lengths of AE and EC to get the length of AF. This is

$$\frac{1}{2} + \frac{\sqrt{5}}{2} \quad \text{or} \quad \frac{1+\sqrt{5}}{2}$$

If we use a hand calculator, we can determine that the decimal equivalent is approximately 1.618. This ratio of the length to the width of the Golden Rectangle is commonly called "Phi" or the Golden Ratio. Phi is the initial letter of the Greek name Phidias.

The number Phi is sometimes called the "Divine Proportion" and is often found in architecture and nature. A common illustration of this is the front of the Parthenon in Athens, Greece, built in the fifth century B.C. Its dimensions fit almost exactly into a Golden Rectangle.

The square extended into a Golden Rectangle also has another interesting feature. If we cut off square ABCD (shown in Fig. 3) we are left with another Golden Rectangle, FGCB! In fact, if wc cut the square from any Golden Rectangle, a new smaller Golden Rectangle is formed. We can continue to cut squares from the next Golden Rectangle and form a new Golden Rectangle each time.

If we find the center of each of these squares and connect them with a smooth curve, we get the logarithimic spiral which is so commonly used in ornamental iron scroll work (Fig. 4-A). This is the same spiral that is found in the well known sea shell, the chambered nautilus. It is often said that when blacksmith design ideas are needed, look at nature. This is really more than coincidence. Mathematical design is seen often in many areas of nature, as we will find later in this article.

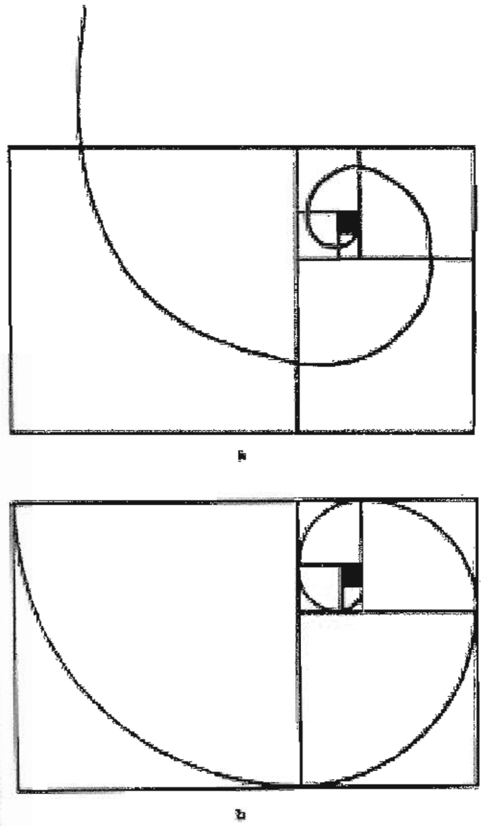

Fig. 4

A way to construct an approximation of the logarithmic spiral is to use one vertex of the square and the length of a side of the square as a radius, then draw a diagonal arc (as shown in Fig. 4-B). By properly choosing the center and changing the radius for each square, the spiral can be constructed in our Golden Rectangle.

Phi is a very curious number. It is the only number that, when subtracting I from it, is its own reciprocal! Thus, Phi - I is]/Phi. Another way of saying this is that 1.618 - I = .618, is also 1/1.618. This can help us in designing the most pleasing rectangular sign. If we know the width, we can find the length by multiplying it by 1.618, as we did in the beginning of the article. Conversely, if we know the length, we can find the width by multiplying by one less, or .618! Phi - I (or .618) is usually called Phi, or Phi Prime, and is often used instead of Phi in computations and discussions.

Readers might remember from their geometry studies, that they were asked to find a point on a line which divided it into mean and extreme

Fig. 5

ratios. That is, to find a point on a line which divides the line so that the ratio of the whole line segment to the longer segment, is in the same ratio as the Ionger segment is to the shorter line segment (see Fig. 5). The ratios (a + b)/a and a/b are both Phi!

From Fig. 5 we could make two Golden Rectangles. One would have length (a + b) and width a, while the smaller one would have length a and width b. You might notice that if you draw the figure, the smaller Golden Rectangle is the one left if you cut the square from the larger Golden Rectangle. Thus, in Fig. 3, a is AB and b is BF, with point P at B.

Phi pops up again in the "Golden Section" of the regular pentagon, or five-sided figure. If we draw diagonals of the regular pentagon, they divide each other in the mean and extreme ratio, or Phi (see Fig. 6). Diagonals AB and CD intersect at P and divide each other in the Golden Ratio, Phi. There are other Phi relationships that the pentagon exhibits. If all the diagonals are drawn, forming the pentacrest, it becomes the secret symbol of the ancient Pythagorean Society.

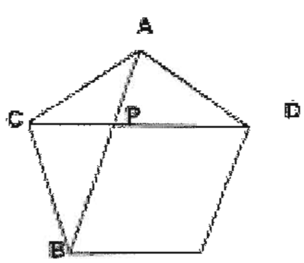

Fig. 6

Phi turns up in some unexpected places. In ancient times Leonardo of Pisa (now called Fibonacci) was working with a series of numbers. Starting with 1, 1, he added the two numbers to get the next number, so 1 +1 = 2. The next one became I + 2 = 3, then came 2 + 3 ~ 5, and so on to get the series: 1, 1, 2, 3, 5, 8, 13, 21, 34, etc. Obviously, the next number will be 21 + 34, = 55.

Adjacent pairs of these numbers (or those next to each other) are often found in nature. For example, the pair 5 to 8 can be found by counting the number of spirals on a pine cone clockwise and counterclockwise.

Pineapple spirals appear in the ratio of 8 to 13.

If we take pairs of adjacent Fibonacci numbers and express them as a fraction, and use a hand calculator to divide and find decimal equivalents for them, an interesting thing happens. As we continue with larger and larger pairs, our decimal quotient gets closer and closer to .618 (see Fig. 7). Here is Phi again!

Mathematicians have proven that as the Fibonacci numbers get larger and larger, the ratio of two adjacent numbers approaches Phi as a limit. This can help us if we are working on a design project and forget the number Phi. We can estimate Phi as closely as we like. For example, 112 is not a very good approximation of Phi, but 2/3 is better. The ratio 3/5 is better still and 5/8 is even closer to Phi. We can go up the sequence as far as we like to estimate Phi as closely as we like! That may be why small flags are commonly 3 x 5, and larger flags 8 x 13.

Most of these relationships are developed in a beautiful paperback book published by Dover. The Divine Proportion, A Study of Mathematical Beauty, by H. F. Huntley, is packed with interesting ideas presented in various levels of mathematical sophistication.

Blacksmithing design, mathematics, and nature are very closely eonnected. Good design can integrate ideas from mathematics and nature to make the most aesthetic ornamental iron pieces.